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INSCRIBING REGULAR PENTAGONS AND DECAGONS.

By Joseph Bowden.

The chief object of the following paper is to prove exact a construction for the regular pentagon that is found in books on mechanical drawing* and is on that account thought by some to be inexact.

Let OA (Fig. 1) be a radius of the circle.

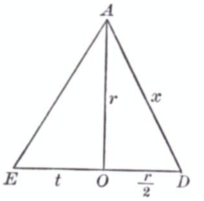

Fig. 1.

Draw OD perpendicular to OA and equal to half the radius. Draw DA and prolong DO to E so that $DE = DA$. Join AE.

Then EO and AE are respectively equal to the sides of the regular inscribed decagon and pentagon.

To prove this let $OA = r$, $DA = x$, $EO = t$.

Then $OD = r/2$. Hence $x^2 = r^2 + (r/2)^2 = 5r^2/4$.

Also $x = AD = ED = EO + OD = t + r/2$.

Therefore $5r^2/4 = x^2 = t^2 + rt + r^2/4$.

Hence

$$r^2 = t^2 + rt,$$
$$t^2 = r^2 - rt = r(r - t),$$
$$r/t = t/(r - t).$$

Now, in the triangle AOD, $x > r/2$.

Therefore, since $t = x - r/2$, t is positive.

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* See Anson K. Cross, "Mechanical Drawing," Boston, Ginn & Co., p. 16; William F. Willard, "A Practical Course in Mechanical Drawing," Chicago, Popular Mechanics Co., p. 26.

Also, since $t^2=r(r-t)$, $r-t$ is positive, and therefore $r>t$.

Therefore, since $r/t=t/(r-t)$, $t>r-t$ and t is equal to the greater segment of the radius when the latter is divided in extreme and mean ratio.

Therefore t is the side of the regular inscribed decagon.

Now

$$\frac{t}{r}=\frac{r-t}{t}=\frac{r}{t}-1.$$

Hence

$$\frac{r}{t}-\frac{t}{r}=1.$$

Squaring,

$$\frac{r^2}{t^2}-2+\frac{t^2}{r^2}=1,$$

$$\frac{r^2}{t^2}+\frac{t^2}{r^2}=3.$$

Hence

$$\frac{t^2}{r^2}=3-\frac{r^2}{t^2}.$$

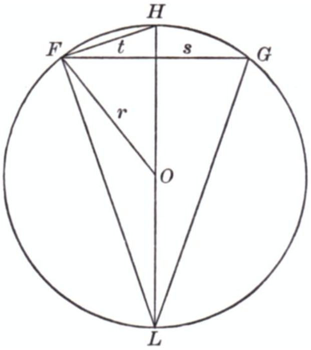

FIG. 2.

Now suppose that FG (Fig. 2) is a side of the regular inscribed pentagon and HL a diameter perpendicular to FG, cutting the smaller of the arcs subtended by FG in H. Let $FG=s$.

Then FH is a side of the regular inscribed decagon.

Now $FL^2 = HL^2 - FH^2 = (2r)^2 - t^2 = 4r^2 - t^2$.

Also, since the triangles FLG and FOH are similar,

$$FG : FH = FL : FO, \text{ or } s : t = FL : r.$$

Hence

$$\frac{s^2}{t^2} = \frac{FL^2}{r^2} = \frac{4r^2 - t^2}{r^2} = 4 - \frac{t^2}{r^2} = 4 - \left(3 - \frac{r^2}{t^2}\right) = 1 + \frac{r^2}{t^2}.$$

Therefore

$$s^2 = r^2 + t^2.$$

Now, in Fig. 1, $AE^2 = r^2 + t^2$. Hence $AE^2 = s^2$ and $AE = s$. Therefore AE is the side of the regular inscribed pentagon.

We see also, incidentally, that *if the radius and the side of the regular inscribed decagon are taken as the perpendicular sides of a right triangle, the hypotenuse will equal the side of the regular inscribed pentagon.**

The above proof does not involve radicals. The reader may be interested to work out one using radicals.

If the quadratic equation $r^2 = t^2 + rt$ is solved for t, we get, since t must be positive, $t = \frac{1}{2}r(\sqrt{5} - 1)$.

If this value is substituted for t in the equality $s^2 = r^2 + t^2$ and this equation solved for s, we get $s = \frac{1}{2}r\sqrt{10 - 2\sqrt{5}}$.

These formulas for t and s are well known.

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* See Phillips and Fisher's "Elements of Geometry," New York, American Book Co., p. 476, prob. 125.